Multi-period Power System Risk Minimization under Wildfire Disruptions

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Electric Grid - Wildfire Interactions

Power lines are impacted by fires

- Increased risk of flashovers due to smoke;
- Power shutoff for safety of firefighters;
- Direct damage to equipment.

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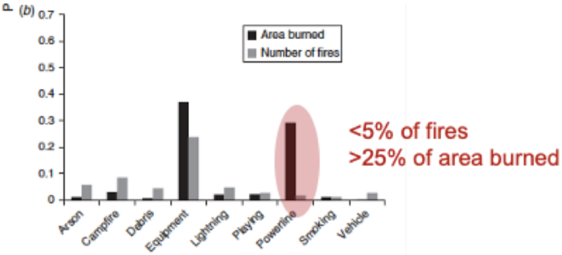
Power lines can ignite fires

- Arcing due to faulty equipment;
- Sparks from downed lines.

Wildfires ignited by power lines are often large



Wildfires in San Diego County. Figure from Syphard, Alexandra D., and Jon E. Keeley. "Location, timing and extent of wildfire vary by cause of ignition." *International Journal of Wildland Fire* 24.1 (2015): 37-47.

- Fires ignited by power lines tend to be **larger** than other fires;
- High wind = higher probability of power line ignitions + higher probability of large fires.

Reducing Wildfire Risk

Measures to reduce the probability of ignition:

1. Undergrounding of lines
2. Vegetation management
3. New equipment
4. Public safety power shutoffs

Method	Advantage	Disadvantage
1	Reduce risk to zero	Costly
2	Quicker to implment	Risk remains, costly
3	Quicker to implment	Risk remains, outages
4	Reduce risk to zero, quick to implment	High societal cost

Interactions between Wildfires and Power Systems

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     $\rightsquigarrow$ Highly effective, but you have to be careful.

Wildfire Disruptions

Endogenous Fire:

- Fire ignited by an energized power grid equipment;
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- Fire ignited by other causes;
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Exogenous Fire

- Fire ignited by other causes;
- Fire can spread and damage power grid equipment;

$\rightsquigarrow$ Grid Operator has no control over risk!

Tradeoffs: Should a power line be de-energized?

	Advantages	Disadvantages
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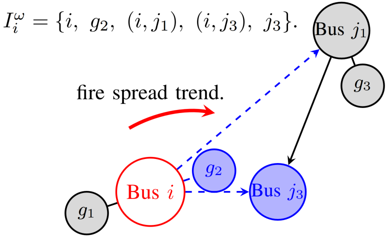
- If too many components are de-energized and an exogenous wildfire occurs, the load-shedding cost will be substantial;
- If an endogenous wildfire occurs, it will affect more components beyond itself.



Two-stage Problem

Consider a multi-period power flow problem
incorporating line de-energization decisions:

- Stochastic Disruptions:

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- Stochastic Disruptions:

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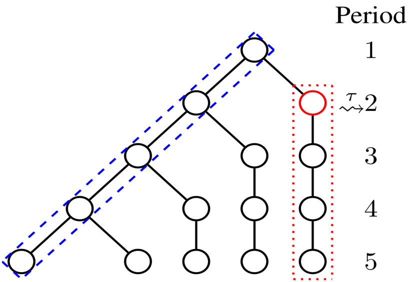
- At most one Disruption:

$\rightsquigarrow$ Modeled as a two-stage stochastic mixed-integer program:

i) First-tage Nominal Plan before a wildfire occurs;

ii) Second-stage Disruption Plan after a disruption occurs.



Two-stage Model

The first-tage decides a nominal plan for the entire horizon:

- Objective is to minimize load shed + the second stage cost;
- This plan is in effect until a wildfire disruption occurs τ^{ω};
- Decisions for line de-energization, dispatch, load shed.

$$Z^* = \min \sum_{\omega \in \Omega} p^{\omega} \left[\sum_{t=1}^{\tau^{\omega}-1} \sum_{d \in \mathcal{D}} w_d (1 - x_{dt}) + f^{\omega}(z_{\cdot \tau^{\omega}-1}, \xi^{\omega}) \right]$$

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The second-stage cost after a disruption occurs:

- Incorporates costs associated with damaged components, new dispatch, and additional load shedding;
- This cost is influenced by the first-stage decision $z_{\cdot \tau^{\omega}-1}$:
  - $z_{\cdot \tau^{\omega}-1}$ represents the shutoff status of components at time $\tau^{\omega} - 1$, just before a disruption occurs.

The First-stage Model

- The first-stage decisions set up a nominal plan which should be carried out until the first wildfire disruption occurs.
  - The first stage make decisions on the entire horizon;
  - The second stage makes decisions after the disruption happens.
- **Objective Function:**

$$\sum_{\omega \in \Omega} p^{\omega} \left[\sum_{t=1}^{\overbrace{\tau^{\omega}-1}^{\text{Before Disruption}}} \underbrace{w_d}_{\text{Priority}} \overbrace{(1 - x_{dt})}^{\text{Unsatisfied Demand}} + \underbrace{f^{\omega}(z_{\tau^{\omega}-1}, \xi^{\omega})}_{\text{Second-stage Cost}} \right].$$

Constraints: DC-OPF

- **Ohm's Law:** $P^L_{ijt} = -b_{ij}(\theta_{it} - \theta_{jt})z_{ijt}$.
  - We linearize it by using big-M method:

$$\overbrace{P^L_{ijt}}^{\text{Real Power}} \leq -b_{ijt}(\theta_{it} - \theta_{jt}) + \overline{\theta}(1 - \overbrace{z_{ijt}}^{\text{Whether (i, j) is functional.}}), \tag{1a}$$

$$P^L_{ijt} \geq -b_{ijt}(\theta_{it} - \theta_{jt}) - \overline{\theta}(1 - z_{ijt}), \tag{1b}$$

$$P^L_{ijt} \leq W_{ij} z_{ijt}, \tag{1c}$$

$$P^L_{ijt} \geq -W_{ij} z_{ijt}. \tag{1d}$$

- **The Generator Limitation:**

$$\underline{\mathsf{P}}^G_g z_{gt} \leq \overbrace{P^G_{gt}}^{\text{Power Generation}} \leq \overline{P}^G_g z_{gt}. \tag{2}$$

- **Nodal Power Balance:**

$$\sum_{g \in \mathcal{G}_i} P^G_{gt} + \sum_{(i,j) \in \mathcal{L}} P^L_{ijt} - \sum_{(j,i) \in \mathcal{L}} P^L_{jit} = \sum_{d \in \mathcal{D}} \overbrace{x_{dt}}^{\text{Satisfied Demand Percentage}} D_{dt}, \quad i \in \mathcal{B}. \tag{3}$$

Constraints: Interactions

- **Component Interactions:** generators, loads, and lines can only be energized if the buses connected with them are energized:

$$z_{it} \geq x_{dt}, \quad i \in \mathcal{B},\ d \in \mathcal{D}_i,\ t = 1, 2, \ldots, T, \tag{4a}$$

$$z_{it} \geq z_{gt}, \quad i \in \mathcal{B},\ g \in \mathcal{G}_i,\ t = 1, 2, \ldots, T, \tag{4b}$$

$$z_{it} \geq z_{ijt}, \quad i \in \mathcal{B},\ (i, j) \in \mathcal{L},\ t = 1, 2, \ldots, T, \tag{4c}$$

$$z_{it} \geq z_{jit}, \quad i \in \mathcal{B},\ (j, i) \in \mathcal{L},\ t = 1, 2, \ldots, T. \tag{4d}$$

- **Logic Constraints:**

$$z_{it} \geq z_{i,t+1}, \quad i \in \mathcal{B}, t = 1, 2, \ldots, T-1, \tag{5a}$$

$$z_{gt} \geq z_{g,t+1}, \quad g \in \mathcal{G}, t = 1, 2, \ldots, T-1, \tag{5b}$$

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First-stage Model

$$Z^* = \min \sum_{\omega \in \Omega} p^\omega \left[\sum_{t=1}^{\tau^\omega - 1} \sum_{d \in \mathcal{D}} w_d(1 - x_{dt}) + f^\omega(z_{\cdot \tau^\omega - 1}, \xi^\omega) \right]$$

s.t. $\forall t \in \mathcal{T}$:

$$P^L_{ijt} \leq -b_{ij} \left(\theta_{it} - \theta_{jt} + \bar{\theta}(1 - z_{ijt})\right) \quad \forall (i,j) \in \mathcal{L} \quad \text{(1a)}$$

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$$\underline{P}^G_g z_{gt} \leq P^G_{gt} \leq \bar{P}^G_g z_{gt} \quad \forall g \in \mathcal{G} \quad \text{(1e)}$$

$$z_{it} \geq x_{dt} \quad \forall i \in \mathcal{B}, d \in \mathcal{D}_i \quad \text{(1f)}$$

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$$z_{it} \geq z_{ijt} \quad \forall i \in \mathcal{B}, (i,j) \in \mathcal{L} \quad \text{(1h)}$$

$$z_{it} \geq z_{jit} \quad \forall i \in \mathcal{B}, (j,i) \in \mathcal{L} \quad \text{(1i)}$$

$$z_{ct} \geq z_{c\,\min\{t+1,T\}} \quad \forall c \in \mathcal{C} \quad \text{(1j)}$$

$$z_{ct} \in \{0,1\} \quad \forall c \in \mathcal{C}. \quad \text{(1k)}$$

Objective function

- Minimize load-shedding cost;
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- Minimize load-shedding cost;
- Minimize second-stage cost.

Power systems operations with shut-offs:

- Linearized Power Flow;
- Power Balance;
- Generator limits.

Logic Constraints:

- Connection to de-energized components;
- No restoration of de-energized components (1j).

Second-stage Problem

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- Effects from a wildfire disruption occurring
  - Minimize load-shedding cost + damage cost from wildfire;
  - Starts at the time of the disruption τ^ω;
  - Subject to de-energization decisions $z_{\cdot\tau^\omega-1}$ made in the first stage.

Exogenous Fire Constraints

- In the event of a wildfire, fire damage will occur

$$\overbrace{\nu_i^\omega}^{\text{Whether damage incurred.}} \geq \underbrace{v_i^\omega}_{\text{Whether i is shut off due to wildfire.}}.$$

- Damaged components will not be energized

$$y_i^\omega \leq 1 - \nu_i^\omega.$$

- The shut-off components stay down after the fire disruption is revealed

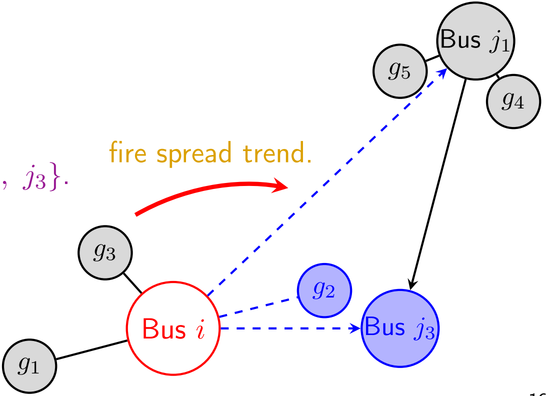
$$y_i^\omega \leq \hat{z}_{i,\tau^\omega - 1}.$$

Endogenous Fire Constraints

- If a component i faults and is not shut off, then it will lead to a fire which will spread to the components adjacent to i

$$\nu_j^\omega \geq \underbrace{u_i^\omega}_{\text{Whether i faults.}} \hat{z}_{i,\tau^\omega-1}, \quad j \in \overbrace{I_i^\omega}^{\text{Set of affected components by i.}}$$

$I_i^\omega = \{i,\ g_2,\ (i,j_1),\ (i,j_3),\ j_3\}.$



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$$\min \quad \sum_{t=\tau^\omega}^{T} \sum_{d\in\mathcal{D}} w_d(1-x^\omega_{dt}) + \sum_{c\in\mathcal{C}} r_c \nu^\omega_c$$

s.t. $\forall t \in \{\tau^\omega, \dots, T\}$:

$$\begin{aligned}
&P^{L,\omega}_{ijt} \le -b_{ij}\left(\theta^\omega_{it} - \theta^\omega_{jt} + \bar{\theta}(1-y^\omega_{ij})\right) && \forall (i,j)\in\mathcal{L} \quad (2a)\\
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&-W_{ij}y^\omega_{ij} \le P^{L,\omega}_{ijt} \le W_{ij}y^\omega_{ij} && \forall (i,j)\in\mathcal{L} \quad (2c)\\
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&\underline{P}^{G,\omega}_g y^\omega_g \le P^{G,\omega}_{gt} \le \bar{P}^{G,\omega}_g y^\omega_g && \forall g\in\mathcal{G} \quad (2e)\\
&y^\omega_i \ge x^\omega_{dt} && \forall i\in\mathcal{B}, d\in\mathcal{D}_i \quad (2f)\\
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&y^\omega_c \le z^\omega_c && \forall c\in\mathcal{C} \quad (2j)\\
&y^\omega_c \le 1-\nu^\omega_c && \forall c\in\mathcal{C} \quad (2k)\\
&\nu^\omega_c \ge v^\omega_c && \forall c\in\mathcal{C} \quad (2l)\\
&\nu^\omega_k \ge u^\omega_c z^\omega_c && \forall c\in\mathcal{C}, k\in I^\omega_c \quad (2m)\\
&z^\omega_c = \hat{z}_{c\tau^\omega-1} && \forall c\in\mathcal{C} \quad (2n)\\
&y^\omega_c, \nu^\omega_c, z^\omega_c \in \{0,1\} && \forall c\in\mathcal{C}. \quad (2o)
\end{aligned}$$

Objective function

- Minimize load-shedding cost;
- Minimize damage cost.

Second-stage Model

$$\min \quad \sum_{t=\tau^\omega}^{T} \sum_{d\in\mathcal{D}} w_d(1-x_{dt}^\omega) + \sum_{c\in\mathcal{C}} r_c \nu_c^\omega$$

$$\text{s.t.} \quad \forall t \in \{\tau^\omega, \dots, T\}:$$

$$\begin{aligned}
&P_{ijt}^{L,\omega} \le -b_{ij}\left(\theta_{it}^\omega - \theta_{jt}^\omega + \bar{\theta}(1-y_{ij}^\omega)\right) && \forall (i,j)\in\mathcal{L} && \text{(2a)}\\
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&\sum_{g\in\mathcal{G}_i} P_{gt}^{G,\omega} + \sum_{(i,j)\in\mathcal{L}_i} P_{ijt}^{L,\omega} = \sum_{d\in\mathcal{D}_i} D_{dt}x_{dt}^\omega && \forall i\in\mathcal{B} && \text{(2d)}\\
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&y_c^\omega \le 1-\nu_c^\omega && \forall c\in\mathcal{C} && \text{(2k)}\\
&\nu_c^\omega \ge v_c^\omega && \forall c\in\mathcal{C} && \text{(2l)}\\
&\nu_k^\omega \ge u_c^\omega z_c^\omega && \forall c\in\mathcal{C}, k\in I_c^\omega && \text{(2m)}\\
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&y_c^\omega, \nu_c^\omega, z_c^\omega \in \{0,1\} && \forall c\in\mathcal{C}. && \text{(2o)}
\end{aligned}$$

Objective function

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- Minimize damage cost.

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- Linearized Power Flow;
- Power Balance;
- Generator limits.

Second-stage Model

$$\min \quad \sum_{t=\tau^\omega}^{T} \sum_{d\in\mathcal{D}} w_d(1-x_{dt}^\omega) + \sum_{c\in\mathcal{C}} r_c \nu_c^\omega$$

s.t. $\forall t \in \{\tau^\omega, \ldots, T\}$:

$$
\begin{aligned}
& P_{ijt}^{L,\omega} \le -b_{ij}\left(\theta_{it}^\omega - \theta_{jt}^\omega + \bar{\theta}(1-y_{ij}^\omega)\right) && \forall (i,j)\in\mathcal{L} && \text{(2a)}\\
& P_{ijt}^{L,\omega} \ge -b_{ij}\left(\theta_{it}^\omega - \theta_{jt}^\omega + \underline{\theta}(1-y_{ij}^\omega)\right) && \forall (i,j)\in\mathcal{L} && \text{(2b)}\\
& -W_{ij}y_{ij}^\omega \le P_{ijt}^{L,\omega} \le W_{ij}y_{ij}^\omega && \forall (i,j)\in\mathcal{L} && \text{(2c)}\\
& \sum_{g\in\mathcal{G}_i} P_{gt}^{G,\omega} + \sum_{(i,j)\in\mathcal{L}_i} P_{ijt}^{L,\omega} = \sum_{d\in\mathcal{D}_i} D_{dt}x_{dt}^\omega && \forall i\in\mathcal{B} && \text{(2d)}\\
& \underline{P}_g^{G,\omega} y_g^\omega \le P_{gt}^{G,\omega} \le \bar{P}_g^{G,\omega} y_g^\omega && \forall g\in\mathcal{G} && \text{(2e)}\\
& y_i^\omega \ge x_{dt}^\omega && \forall i\in\mathcal{B}, d\in\mathcal{D}_i && \text{(2f)}\\
& y_i^\omega \ge y_g^\omega && \forall i\in\mathcal{B}, g\in\mathcal{G}_i && \text{(2g)}\\
& y_i^\omega \ge y_{ij}^\omega && \forall i\in\mathcal{B}, (i,j)\in\mathcal{L} && \text{(2h)}\\
& y_i^\omega \ge y_{ji}^\omega && \forall i\in\mathcal{B}, (j,i)\in\mathcal{L} && \text{(2i)}\\
& y_c^\omega \le z_c^\omega && \forall c\in\mathcal{C} && \text{(2j)}\\
& y_c^\omega \le 1-\nu_c^\omega && \forall c\in\mathcal{C} && \text{(2k)}\\
& \nu_c^\omega \ge v_c^\omega && \forall c\in\mathcal{C} && \text{(2l)}\\
& \nu_k^\omega \ge u_c^\omega z_c^\omega && \forall c\in\mathcal{C}, k\in I_c^\omega && \text{(2m)}\\
& z_c^\omega = \hat{z}_{c\tau^\omega-1} && \forall c\in\mathcal{C} && \text{(2n)}\\
& y_c^\omega, \nu_c^\omega, z_c^\omega \in \{0,1\} && \forall c\in\mathcal{C}. && \text{(2o)}
\end{aligned}
$$

Objective function

- Minimize load-shedding cost;
- Minimize damage cost.

Power systems operations with shut-offs:

- Linearized Power Flow;
- Power Balance;
- Generator limits.

Logic Constraints:

- Exo. damage (2l);
- De-energ. comp. No End. risk (2m);
- End. fire spread (2m);
- Damaged components become de-energized (2k).

Decomposition Algorithm - Master Problem

Substitute the second-stage value function with a cutting-plane approximation:

$$
\begin{aligned}
(M) \quad \min \quad & \sum_{\omega \in \Omega} p^{\omega} \left[\sum_{t=1}^{\tau^{\omega}-1} w_d (1 - x_{dt}) + V^{\omega} \right] \\
\text{s.t.} \quad & (1a) - (1k), \\
& V^{\omega} \geq v^{\omega,\ell} + (\lambda^{\omega,\ell})^{\top} (z_{\cdot \tau^{\omega}-1} - \hat{z}_{\cdot \tau^{\omega}-1}), \quad \ell = 1, \ldots, L, \omega \in \Omega.
\end{aligned}
$$

Decomposition Algorithm - Cut Generation Problem

To generat tight cuts, especially, Lagrangian cuts, solve the following non-anticipativity dual problem

$$\max \quad R^\omega(\hat{z}, \lambda),$$

where $R^\omega(\hat{z}, \lambda)$ is the Lagrangian relaxation problem

$$\begin{aligned} R^\omega(\hat{z}, \lambda) = \min \quad & \texttt{second-stage obj} + \lambda^\top (z^\omega - \hat{z}^\omega) \\ \text{s.t.} \quad & \texttt{second-stage constraints}^1 \\ & z^\omega \in \{0,1\}^{|\mathcal{C}|} \end{aligned}$$

[1] except non-anticipativity constraint

Decomposition Algorithm - SMC

To generat tight cuts, especially, Lagrangian cuts, solve the following non-anticipativity dual problem

$$\max \quad R^{\omega}(\hat{z}, \lambda).$$

We solve the following problem

$$\begin{aligned} \min \quad & \lambda^{\top}\lambda \\ \text{s.t.} \quad & R^{\omega}(\hat{z}, \lambda) \geq (1-\delta) f^{\omega}(\hat{z}_{\cdot,\tau^{\omega}}, \xi^{\omega}). \end{aligned}$$

Test System: RTS-GMLC

73-bus test system with geographic coordinates:

- Three levels of load priority and damage cost to generations;
- Visualization of "Scenarios".

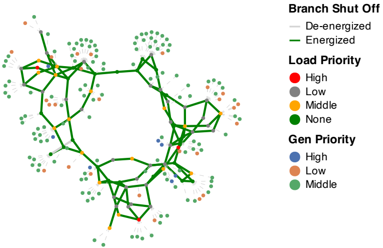

Figure: Power System Configuration.

Test System: RTS-GMLC

73-bus test system with geographic coordinates:

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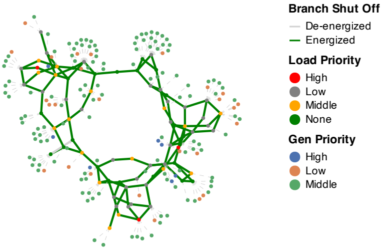


Figure: Power System Configuration.

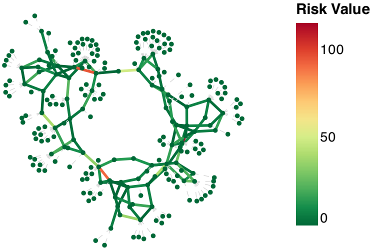
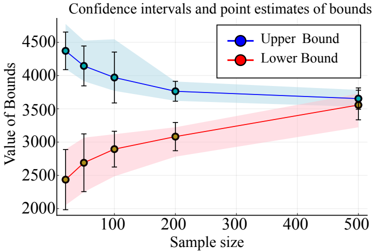


Figure: Wildfire risk value at $t = 13$.

Solution Quality

Sample size of 500 allows for sufficient convergence of upper and lower bounds:



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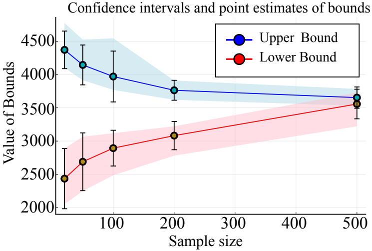


Table: Run-time (sec.) of utilizing SMC and LC with a tolerance of 1.0%, compared with a run-time of solving `extensive formulation` by Gurobi with MIPGap $= 1\%$.

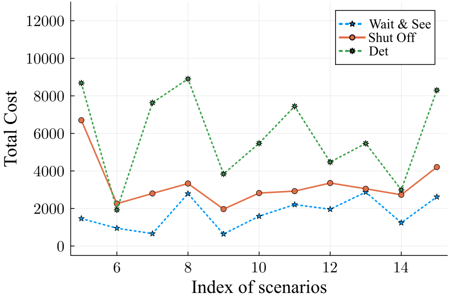
Sample size	Algorithm Time SMC (Sec.)	Algorithm Time LC (Sec.)	Gurobi Time (Sec.)
20	712	3287	84
50	1248	8741	376
100	2604	17911	1510
200	5323	> 18000	9186
500	12776	> 18000	> 18000

This shows:

- Convergence;
- Computational efficiency of the enhanced Lagrangian cuts (SMC).

Benchmarks

- Benchmark Comparison
  - Wait & See (Perfect Information);
  - Deterministic (No Shutoff).
- Evaluation on $5,000$ scenarios
  - Wait & See is 41.6% better than Stochastic Shutoff Plan;
  - No-shutoff is 63.9% worse than Stochastic Shutoff Plan.



Comparison against Det. Shutoff Model

Previous Research without Scenario Utilization[1]:

- Employ wildfire risk as a metric to quantify risk level;
- Focus solely on a deterministic model:

$$\min \quad \sum_{t=1}^{T} \left[\alpha \frac{\overbrace{R_{Fire,t}}^{\text{Wildfire Risk}}}{R_{Tot}} - \underbrace{(1-\alpha)}_{\text{Tradeoff}} \frac{\overbrace{\sum_{d \in \mathcal{D}} x_{dt} w_d D_{dt}}^{\text{Load Delivery}}}{D_{Tot}} \right]$$

$$\text{s.t.} \quad (1a) - (1k)$$

$$R_{Fire,t} = \sum_{c \in \mathcal{C}} R_{ct} z_{ct} \qquad \forall t \in \mathcal{T}.$$

[1] N. Rhodes, L. Ntaimo, and L. Roald, "Balancing Wildfire Risk and Power Outages Through Optimized Power Shut-Offs," Dec. 2020

Comparison against Risk Model

- Many different tradeoffs of risk and load shed are possible
- Solution quality is at least 48.4% worse than the stochastic model.

α	Non-Disr. Load shed	Disruptive Load shed	Disruptive Damage	Total Cost $g_n(\cdot)$	Total Cost RRI
0.0	0.0	3247.0	2814.0	5936.2	63.9%
0.1	148.6	3843.5	1767.5	5495.4	51.7%
0.2	162.9	3850.7	1739.8	5480.8	51.4%
0.3	221.4	3891.0	1713.9	5489.4	51.6%
0.4	171.6	3786.3	1783.4	5454.9	50.6%
0.5	251.1	3712.8	1773.9	5373.7	48.4%
0.6	106.7	3686.8	1817.0	5390.5	48.9%
0.7	469.3	3721.4	1754.1	5425.9	50.0%
0.8	883.6	3949.0	1700.1	5532.7	52.8%
0.9	1505.8	4285.3	1341.1	5510.5	52.2%
X^*	1678.2	2355.2	1333.6	3612.7	0.0%

Acknowledgements

Thanks for Coming!

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