Multistage Stochastic Program for Mitigating Power System Risks under Wildfire Disruptions

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Motivation

- Power supply is critical to our lives.
- Wildfire can cause severe damages in power systems.
  - In California, 10% of wildfires ignited by electrical equipment were responsible for more than 70% of damages.
  - The 2020 Australian bushfires lasted several months and spread across multiple states, affecting numerous power infrastructure components.

Motivation

Challenges in Power System Management:

- Uncertainty: Wildfires can occur at any time and place, creating unpredictable risks;
- Duration and Spread: Wildfires can last for varying durations and have the potential to spread across multiple components.

Importance of Proactive Measures:

- Effective strategies are essential to minimize the impact of wildfires;
- Proactive measures can help in preparing for, responding to, and recovering from wildfire-induced disruptions.

Wildfire Disruptions

Exogenous Fire

- Ignited by external factors (e.g., human activities, lightning);
- Can spread and damage power grid equipment;

$\rightsquigarrow$ Grid operator has **no control** over risk!

Wildfire Disruptions

Exogenous Fire

- Ignited by external factors (e.g., human activities, lightning);
- Can spread and damage power grid equipment;

$\rightsquigarrow$ Grid operator has **no control** over risk!

Endogenous Fire

- Ignited by energized power grid equipment;
- Can spread and damage other grid equipment;

$\rightsquigarrow$ Risk can be reduced by **turning off** power lines.

Tradeoffs: Should a power line be de-energized?

	Advantages	Disadvantages
De-energized	Prevents ignition	Cannot deliver power
Energized	Power delivery	Cannot prevent ignition

The disadvantages are exacerbated in the event of a disruption:

$\rightsquigarrow$ **De-energization** + **Exogenous** $\Rightarrow$ Blackout.

- If too many components are de-energized, an exogenous wildfire can lead to significant load-shedding due to the inability to supply power.

$\rightsquigarrow$ **Energization** + **Failure** $\Rightarrow$ Endogenous.

- If components remain energized and an endogenous wildfire occurs, it can spread to other components, causing widespread damage.

Multistage Model

Consider a multi-period power flow problem incorporating line de-energization decisions:

- Stochastic Disruptions:
  - $\rightsquigarrow$ Timing, location, and magnitude of the wildfires are uncertain.

Multistage Model

Consider a multi-period power flow problem incorporating line de-energization decisions:

- Stochastic Disruptions:
  - ⇝ Timing, location, and magnitude of the wildfires are uncertain.
- Modeling:
  - ⇝ Modeled as a multistage stochastic mixed-integer program:
    - **First-stage Nominal Plan**: Before any wildfire occurs.
    - t-**stage Disruption Plan**: Before the t-th disruption occurs.

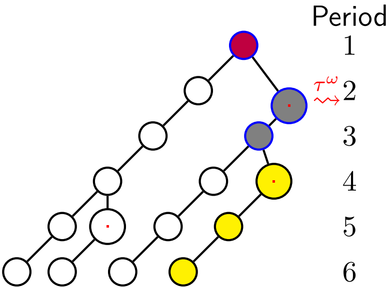


Figure: The presence of a "." symbol within the nodes denotes the occurrence of a disruption.

First-stage Model – Objective Function

The first-tage model decides a nominal plan for the entire horizon:

- Objective is to minimize load shed + the second stage cost;
- This plan is in effect until the first disruption occurs $\tau^{\dot{\omega}}$;
- Decisions for line de-energization, dispatch, load shed.

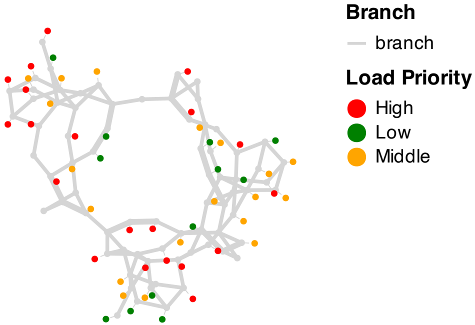
$$\sum_{\dot{\omega} \in \Omega|_{\omega_0}} p^{\dot{\omega}} \left[\sum_{t=1}^{\overbrace{\tau^{\dot{\omega}} - 1}^{\text{Before Disruption}}} \sum_{d \in \mathcal{D}} w_d s_{dt} + \underbrace{f^{\dot{\omega}}(z_{\tau^{\dot{\omega}}-1})}_{\text{Second-stage Cost}} \right].$$

First-stage Model – Fairness

The burden of load-shedding should not:

- Fall solely on lower-priority loads;
- Be evenly distributed.

Require better tradeoffs!



First-stage Model – Constraints

$$
\begin{aligned}
& P^L_{ijt} \le -b_{ij}\left(\theta_{it} - \theta_{jt} + \bar{\theta}(1 - z_{ijt})\right) && \forall (i,j) \in \mathcal{L} && \text{(1a)}\\
& P^L_{ijt} \ge -b_{ij}\left(\theta_{it} - \theta_{jt} + \underline{\theta}(1 - z_{ijt})\right) && \forall (i,j) \in \mathcal{L} && \text{(1b)}\\
& -W_{ij} z_{ijt} \le P^L_{ijt} \le W_{ij} z_{ijt} && \forall (i,j) \in \mathcal{L} && \text{(1c)}\\
& \sum_{g \in \mathcal{G}_i} P^G_{gt} + \sum_{l \in \mathcal{L}_i} P^L_{lt} = \sum_{d \in \mathcal{D}_i} D_{dt}(1 - s_{dt}) && \forall i \in \mathcal{B} && \text{(1d)}\\
& \underline{P}^G_g z_{gt} \le P^G_{gt} \le \bar{P}^G_g z_{gt} && \forall g \in \mathcal{G} && \text{(1e)}\\
& z_{it} \ge x_{dt} && \forall i \in \mathcal{B}, d \in \mathcal{D}_i && \text{(1f)}\\
& z_{it} \ge z_{gt} && \forall i \in \mathcal{B}, g \in \mathcal{G}_i && \text{(1g)}\\
& z_{it} \ge z_{lt} && \forall i \in \mathcal{B}, l \in \mathcal{L}_i && \text{(1h)}\\
& r_{ct} \ge r_{c,\max\{t-1,1\}} && \forall c \in \mathcal{C} && \text{(1i)}\\
& r_{ct} - r_{c,\max\{t-1,1\}} \ge z_{ct} - z_{c,\max\{t-1,1\}} && \forall c \in \mathcal{C} && \text{(1j)}\\
& \sum_{t \in \mathcal{T}} (s_{dt} - s_{d't}) \le \beta \cdot T && \forall d, d' \in \mathcal{D} && \text{(1k)}\\
& z_{ct}, r_{ct} \in \{0, 1\} && \forall c \in \mathcal{C}. && \text{(1l)}
\end{aligned}
$$

Power systems operations with shut-offs:

- Linearized Power Flow;
- Power Balance and Limits;

Logic Constraints:

- Connection to de-energized components;
- Restoration can occur only once.

First-stage Model – Constraints

$$
\begin{aligned}
& P_{ijt}^L \le -b_{ij}\left(\theta_{it} - \theta_{jt} + \bar{\theta}(1 - z_{ijt})\right) && \forall (i,j) \in \mathcal{L} \quad \text{(1a)}\\
& P_{ijt}^L \ge -b_{ij}\left(\theta_{it} - \theta_{jt} + \underline{\theta}(1 - z_{ijt})\right) && \forall (i,j) \in \mathcal{L} \quad \text{(1b)}\\
& -W_{ij} z_{ijt} \le P_{ijt}^L \le W_{ij} z_{ijt} && \forall (i,j) \in \mathcal{L} \quad \text{(1c)}\\
& \sum_{g \in \mathcal{G}_i} P_{gt}^G + \sum_{l \in \mathcal{L}_i} P_{lt}^L = \sum_{d \in \mathcal{D}_i} D_{dt}(1 - s_{dt}) && \forall i \in \mathcal{B} \quad \text{(1d)}\\
& \underline{P}_g^G z_{gt} \le P_{gt}^G \le \bar{P}_g^G z_{gt} && \forall g \in \mathcal{G} \quad \text{(1e)}\\
& z_{it} \ge x_{dt} && \forall i \in \mathcal{B}, d \in \mathcal{D}_i \quad \text{(1f)}\\
& z_{it} \ge z_{gt} && \forall i \in \mathcal{B}, g \in \mathcal{G}_i \quad \text{(1g)}\\
& z_{it} \ge z_{lt} && \forall i \in \mathcal{B}, l \in \mathcal{L}_i \quad \text{(1h)}\\
& r_{ct} \ge r_{c,\max\{t-1,1\}} && \forall c \in \mathcal{C} \quad \text{(1i)}\\
& r_{ct} - r_{c,\max\{t-1,1\}} \ge z_{ct} - z_{c,\max\{t-1,1\}} && \forall c \in \mathcal{C} \quad \text{(1j)}\\
& \sum_{t \in \mathcal{T}} (s_{dt} - s_{d't}) \le \beta \cdot T && \forall d, d' \in \mathcal{D} \quad \text{(1k)}\\
& z_{ct}, r_{ct} \in \{0,1\} && \forall c \in \mathcal{C}. \quad \text{(1l)}
\end{aligned}
$$

Power systems operations with shut-offs:

- Linearized Power Flow;
- Power Balance and Limits;

Logic Constraints:

- Connection to de-energized components;
- Restoration can occur only once.

Fairness Constraint

- The parameter β is to regulate the level of fairness.

t-stage Model – Objective Function

The t-stage problem $f^{\omega}(\hat{z}^{\omega'}_{\tau^{\omega}-1})$ is an optimization problem:

- ω is the realization of the t-stage disruption;
- $\dot{\omega}$ is the realization of the $(t+1)$-stage disruption;
- The $(t-1)$-stage state decision at time $\tau^{\omega}-1$ serving as its input;

$$\min \sum_{\dot{\omega}\in\Omega|_{\omega}} p^{\dot{\omega}} \left[\sum_{t=\tau^{\omega}}^{\tau^{\dot{\omega}}-1} \sum_{d\in\mathcal{D}} w_d s^{\omega}_{dt} + \sum_{c\in\mathcal{C}} \underbrace{c^r_c \nu^{\omega}_c}_{\text{Damage cost}} + f^{\dot{\omega}}(z^{\omega}_{\tau^{\dot{\omega}}-1}) \right].$$

- Effects from a wildfire disruption occurring
  - min load shed + damage cost from wildfire + future costs;
  - Starts at the time of the disruption τ^{ω};
  - s.t. de-energization decisions $z_{\cdot\tau^{\omega}-1}$ made in the $(t-1)$ stage.

t-stage Model – Constraints

$$
\begin{aligned}
& P^{L,\omega}_{ijt} \le -b_{ij}\left(\theta^\omega_{it} - \theta^\omega_{jt} + \bar{\theta}(1 - z^\omega_{ijt})\right) && \forall (i,j) \in \mathcal{L} \quad \text{(2a)}\\
& P^{L,\omega}_{ijt} \ge -b_{ij}\left(\theta^\omega_{it} - \theta^\omega_{jt} + \underline{\theta}(1 - z^\omega_{ijt})\right) && \forall (i,j) \in \mathcal{L} \quad \text{(2b)}\\
& -W_{ij} z^\omega_{ijt} \le P^{L,\omega}_{ijt} \le W_{ij} z^\omega_{ijt} && \forall (i,j) \in \mathcal{L} \quad \text{(2c)}\\
& \sum_{g \in \mathcal{G}_i} P^{G,\omega}_{gt} + \sum_{l \in \mathcal{L}_i} P^{L,\omega}_{lt} = \sum_{d \in \mathcal{D}_i} D_{dt}(1 - s^\omega_{dt}) && \forall i \in \mathcal{B} \quad \text{(2d)}\\
& \underline{P}^{G,\omega}_g z^\omega_{gt} \le P^{G,\omega}_{gt} \le \bar{P}^{G,\omega}_g z^\omega_{gt} && \forall g \in \mathcal{G} \quad \text{(2e)}\\
& z^\omega_{it} \ge x^\omega_{dt} && \forall i \in \mathcal{B}, d \in \mathcal{D}_i \quad \text{(2f)}\\
& z^\omega_{it} \ge z^\omega_{gt} && \forall i \in \mathcal{B}, g \in \mathcal{G}_i \quad \text{(2g)}\\
& z^\omega_{it} \ge z^\omega_{lt} && \forall i \in \mathcal{B}, l \in \mathcal{L}_i \quad \text{(2h)}\\
& z^\omega_{ct-1} \ge z^\omega_{ct} && \forall c \in \mathcal{C} \quad \text{(2i)}\\
& z^\omega_{ct} \le 1 - \nu^\omega_c && \forall c \in \mathcal{C} \quad \text{(2j)}\\
& \nu^\omega_c \ge v^\omega_c && \forall c \in \mathcal{C} \quad \text{(2k)}\\
& \nu^\omega_k \ge u^\omega_c z^\omega_{c\tau^\omega-1} && \forall c \in \mathcal{C}, k \in I^\omega_c \quad \text{(2l)}\\
& z^\omega_{c\tau^\omega-1} = \hat{z}^{\omega'}_{c\tau^\omega-1} && \forall c \in \mathcal{C} \quad \text{(2m)}\\
& z^\omega_{ct}, \nu^\omega_c, z^\omega_{ct} \in \{0,1\} && \forall c \in \mathcal{C}. \quad \text{(2n)}
\end{aligned}
$$

Power systems operations with shut-offs:

- Linearized Power Flow;
- Power Balance;
- Generator limits.

Logic Constraints:

- Exo. damage (2k);
- End. fire spread (2l);

t-stage Model – Constraints

$$P_{ijt}^{L,\omega} \leq -b_{ij}\left(\theta_{it}^{\omega} - \theta_{jt}^{\omega} + \bar{\theta}(1 - z_{ijt}^{\omega})\right) \qquad \forall (i,j) \in \mathcal{L} \quad \text{(2a)}$$

$$P_{ijt}^{L,\omega} \geq -b_{ij}\left(\theta_{it}^{\omega} - \theta_{jt}^{\omega} + \underline{\theta}(1 - z_{ijt}^{\omega})\right) \qquad \forall (i,j) \in \mathcal{L} \quad \text{(2b)}$$

$$-W_{ij} z_{ijt}^{\omega} \leq P_{ijt}^{L,\omega} \leq W_{ij} z_{ijt}^{\omega} \qquad \forall (i,j) \in \mathcal{L} \quad \text{(2c)}$$

$$\sum_{g \in \mathcal{G}_i} P_{gt}^{G,\omega} + \sum_{l \in \mathcal{L}_i} P_{lt}^{L,\omega} = \sum_{d \in \mathcal{D}_i} D_{dt}(1 - s_{dt}^{\omega}) \qquad \forall i \in \mathcal{B} \quad \text{(2d)}$$

$$\underline{P}_g^{G,\omega} z_{gt}^{\omega} \leq P_{gt}^{G,\omega} \leq \bar{P}_g^{G,\omega} z_{gt}^{\omega} \qquad \forall g \in \mathcal{G} \quad \text{(2e)}$$

$$z_{it}^{\omega} \geq x_{dt}^{\omega} \qquad \forall i \in \mathcal{B}, d \in \mathcal{D}_i \quad \text{(2f)}$$

$$z_{it}^{\omega} \geq z_{gt}^{\omega} \qquad \forall i \in \mathcal{B}, g \in \mathcal{G}_i \quad \text{(2g)}$$

$$z_{it}^{\omega} \geq z_{lt}^{\omega} \qquad \forall i \in \mathcal{B}, l \in \mathcal{L}_i \quad \text{(2h)}$$

$$z_{ct-1}^{\omega} \geq z_{ct}^{\omega} \qquad \forall c \in \mathcal{C} \quad \text{(2i)}$$

$$z_{ct}^{\omega} \leq 1 - \nu_c^{\omega} \qquad \forall c \in \mathcal{C} \quad \text{(2j)}$$

$$\nu_c^{\omega} \geq v_c^{\omega} \qquad \forall c \in \mathcal{C} \quad \text{(2k)}$$

$$\nu_k^{\omega} \geq u_c^{\omega} z_{c\tau^{\omega}-1}^{\omega} \qquad \forall c \in \mathcal{C}, k \in I_c^{\omega} \quad \text{(2l)}$$

$$z_{c\tau^{\omega}-1}^{\omega} = \hat{z}_{c\tau^{\omega}-1}^{\omega'} \qquad \forall c \in \mathcal{C} \quad \text{(2m)}$$

$$z_{ct}^{\omega}, \nu_c^{\omega}, z_{ct}^{\omega} \in \{0,1\} \qquad \forall c \in \mathcal{C}. \quad \text{(2n)}$$

Power systems operations with shut-offs:

- Linearized Power Flow;
- Power Balance;
- Generator limits.

Logic Constraints:

- Exo. damage (2k);
- End. fire spread (2l);

Non-anticipativity Constraint:

- ω' is the realization of the $(t-1)$-stage disruption.

Decomposition Algorithm - First Master Problem

Approximate the value function with cutting-planes:

$$\underline{Z}_\ell^* = \min \quad \sum_{\dot\omega \in \Omega|_{\omega_0}} p^{\dot\omega} \left[\sum_{t=1}^{\tau^{\dot\omega}-1} \sum_{d \in \mathcal{D}} w_d s_{dt} + V^{\dot\omega} \right] \qquad (M_\ell)$$

$$\text{s.t.} \quad \text{Constrictions}(1a) - (1l) \qquad \forall t \in \mathcal{T}$$

$$V^{\dot\omega} \geq (\lambda^{\dot\omega,k})^\top (z_{\tau^{\dot\omega}-1} - \hat{z}^k_{\tau^{\dot\omega}-1}) +$$

$$v^{\dot\omega,k}, \quad \forall \dot\omega \in \Omega|_{\omega_0}, k = 1, \dots, \ell - 1,$$

Decomposition Algorithm - Subsequential Master Problem

Substitute the value function with a cutting-plane approximation:

$$
\begin{aligned}
&\underline{f}^{\omega}_{\ell}(\hat{z}^{\omega',\ell}_{\tau^{\omega}-1}) = \qquad\qquad (S^{\omega}_{\ell})\\
\min \quad & \sum_{\dot{\omega}\in\Omega|_{\omega}} p^{\dot{\omega}} \left[\sum_{t=\tau^{\omega}}^{\tau^{\dot{\omega}}-1} \sum_{d\in\mathcal{D}} w_d s^{\omega}_{dt} + \sum_{c\in\mathcal{C}} c^{r}_{c}\nu^{\omega}_{c} + V^{\dot{\omega}} \right]\\
\text{s.t.} \quad & \text{Constrictions}(2a) - (2l), (2n) \quad \forall t \in \{\tau^{\omega}, \dots, T\}\\
& V^{\dot{\omega}} \geq (\lambda^{\dot{\omega},k})^{\top}(z^{\omega}_{\tau^{\dot{\omega}}-1} - \hat{z}^{\dot{\omega},k}_{\tau^{\dot{\omega}}-1}) + \\
& \qquad\qquad v^{\dot{\omega},k}, \qquad \forall \dot{\omega} \in \Omega|_{\omega}, k = 1, \dots, \ell - 1\\
& z^{\omega}_{\tau^{\omega}-1} = \hat{z}^{\omega',\ell}_{\tau^{\omega}-1}.
\end{aligned}
$$

Decomposition Algorithm - Cut Generation Problem

To generat Lagrangian cuts, we solve the following non-anticipativity Lagrangian dual problem

$$\max \quad \mathcal{R}_\ell^\omega(\hat{z}^{\omega',\ell}, \lambda; \mathcal{Z}),$$

where $\mathcal{R}_\ell^\omega(\cdot,\cdot)$ is the Lagrangian relaxation problem

$$\begin{aligned}
&\mathcal{R}_\ell^\omega(\hat{z}^{\omega',\ell}, \lambda; \mathcal{Z}) = \\
&\min \quad \sum_{\dot{\omega}\in\Omega|_\omega} p^{\dot{\omega}} \left[\sum_{t=\tau^\omega}^{\tau^{\dot{\omega}}-1} \sum_{d\in\mathcal{D}} w_d s_{dt}^\omega + \sum_{c\in\mathcal{C}} c_c^r \nu_c^\omega + V^{\dot{\omega}} \right] + \lambda^\top (\hat{z}_{\tau^\omega-1}^{\omega',\ell} - z_{\tau^\omega-1}^\omega) \\
&\text{s.t.} \quad \text{Constrictions}(2a)-(2l),(2n) \quad \forall t \in \{\tau^\omega,\ldots,T\} \\
&\quad V^{\dot{\omega}} \geq (\lambda^{\dot{\omega},k})^\top (z_{\tau^{\dot{\omega}}-1}^\omega - \hat{z}_{\tau^{\dot{\omega}}-1}^{\dot{\omega},k}) + \\
&\qquad\qquad v^{\dot{\omega},k}, \qquad \forall \dot{\omega} \in \Omega|_\omega, k = 1,\ldots,\ell \\
&\quad z_{c\tau^\omega-1} \in \mathcal{Z} \qquad\qquad \forall c \in \mathcal{C}.
\end{aligned}$$

Decomposition Algorithm - Square-Minimization Cut

- Lagrangian cuts may be steep and fail to provide a good lower approximation at other solutions;
- To address this limitation, we use an alternative cut-generation problem (2):

$$\min_{\lambda} \quad \lambda^\top \lambda \tag{2a}$$

$$\text{s.t.} \quad \mathcal{R}_\ell^\omega(\hat{z}^{\omega',\ell}, \lambda; \mathcal{Z}) \geq (1-\delta)\underline{f}_\ell^\omega(\hat{z}_{\tau^\omega - 1}^{\omega',\ell}), \tag{2b}$$

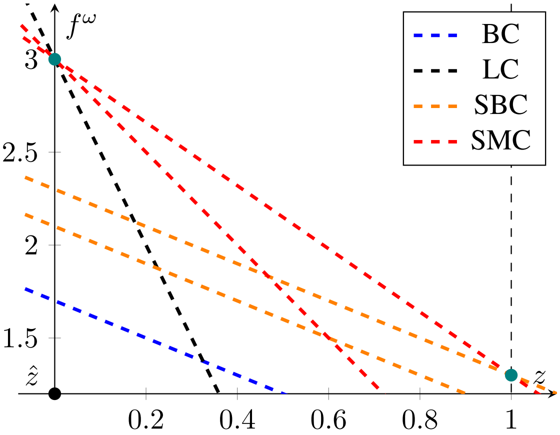


Figure: The upper and lower SBC (SMC) are obtained by taking $\mathcal{Z} = \{0, 1\}$ and $[0, 1]$, respectively.

Test System: RTS-GMLC

73-bus test system with geographic coordinates:

- Three levels of load priority and damage cost to generations;
- Visualization of "Scenarios".

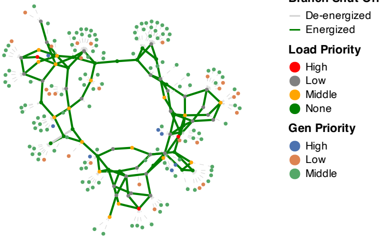


Figure: Power System Configuration.

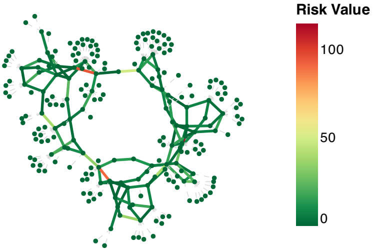


Figure: Wildfire risk value at $t = 13$.

Cut Performance

Table: Assessing Decomp. Alg. performance with different cut families.

Cut Type	Solution Quality			Runtime (sec.)	
	LB	UB	Gap	Time/Iter.	Total
BC	3305.3	3457.8	4.41%	214.2	5377
LC	3306.9	4116.8	19.67%	193.4	3803
SBC^B	3397.5	3449.6	1.51%	237.7	8472
SBC^I	3323.1	3433.6	3.15%	210.8	10500
SMC^B	3414.9	3426.5	0.34%	445.4	20516
SMC^I	3368.6	3419.3	1.48%	391.9	13414

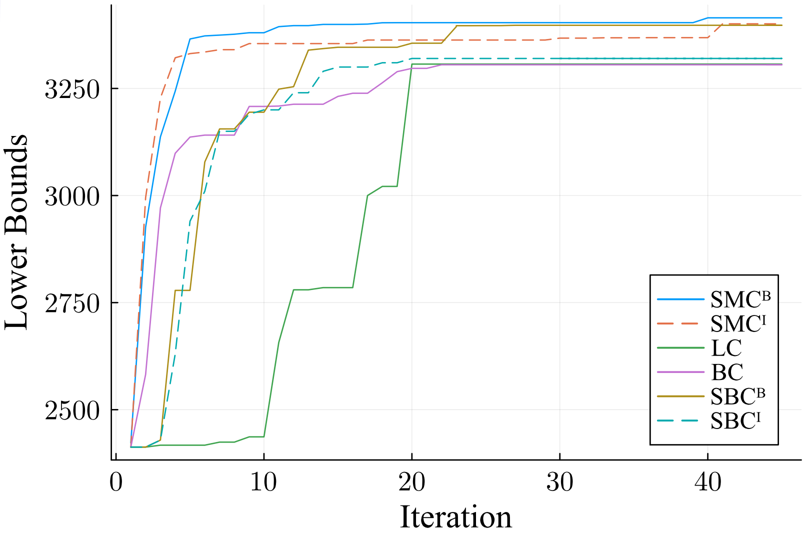


Figure: Convergence v.s. Iter.

Fairness Evaluation: Fairness $\beta = 0.0$ v.s. $\beta = 0.4$

- The maximum load-shedding percentage is at 10% vs 40%.
- The total amount of load shedding is 9.2% vs 4.9%.

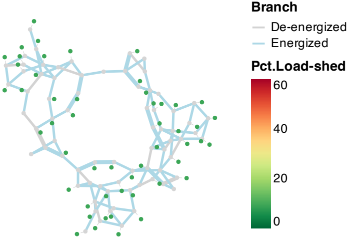


Figure: Load-shedding is evenly distributed.

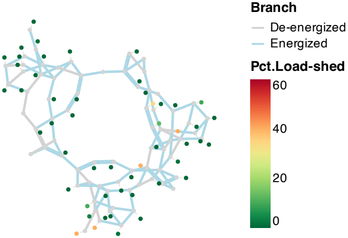


Figure: Load-shedding is concentrated on 4 lower-priority loads.

Restoration v.s. No Restoration

Significant disruptive cost reduction ($> 10\%$).

Setting		Nominal	Disruptive		Total Cost
Res.	β	Load shed	Load shed	Damage	$g(\cdot)$
RES.	0.0	23276.1	1789.0	1389.0	18517.6
	0.4	2978.4	1971.7	1515.6	5948.9
	0.6	2873.5	1967.9	1533.7	5886.4
Non.	0.0	21710.1	2108.8	1957.4	18370.4
	0.4	2933.7	2077.3	1887.3	6409.6
	0.6	2776.7	2045.8	1937.2	6331.7

Acknowledgements

SCAN ME

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